Bousso's Covariant Entropy Bound and Padmanabhan's Emergent Paradigm

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Abstract

The Bousso's covariant entropy conjecture is invariant under time reversal and consequently its origin must be statistical rather than thermodynamical. This may impose a fundamental constraint on the number of degrees of freedom in nature. Indeed, the covariant entropy bound imposes an upper entropy bound for any physical system. Considering a cosmological system, we show that Padmanabhan's emergent paradigm, which indicates that the emergence of cosmic space is due to the discrepancy between the surface and bulk degrees of freedom, leads to a lower entropy bound. The lower and upper entropy bounds may coincide on the apparent horizon for the radiation field and cosmological constant with the equations of state parameters $\omega = \frac{1}{3}$ and $\omega = -1$, respectively. Moreover, the maximal entropy inside the apparent horizon occurs when it is filled completely by the radiation field or cosmological constant. The maximal entropy property is lost at matter dominant era with the equations of state parameter $\omega = 0$. Therefore, one may interpret the current acceleration of the Universe as a tendency of the system of Universe to recover this lost maximal entropy property, at late time dominated by cosmological constant.

1 Introduction

The idea that gravity behaves as an emergent phenomenon is referred to the proposal made by Sakharov in 1967 [2]. In this proposal which is named as the induced gravity, the spacetime background emerges as a mean field approximation of some underlying microscopic degrees of freedom similar to hydrodynamics or continuum elasticity theory from molecular physics [3]. Current research works on the relation between gravitational dynamics and thermodynamics support such a point of view [4]. In this line of activity, the major attention is focused on how the gravitational field equations can be obtained from the thermodynamical point of view. In 1995, the Einstein field equations are obtained in the pioneer work by Jacobson by using the equivalence principle and Clausius relation $dQ = TdS$ where Q, T and S are the heat, temperature and entropy, respectively [5]. The key point is to demand that the Clausius relation holds for the all local Rindler causal horizons with Q and T interpreted as the energy flux and Unruh temperature, as seen by an accelerated observer located inside the horizon. In this regard, the Einstein filed equations are nothing but the equations of state of spacetime. The Clausius relation also arises when one treats the gravitational field equations as an entropy balance law across a null surface, i.e $S_m = S_{grav}$ [6].

Moreover, another viewpoint that the gravity is not a fundamental interaction has been advocated by Verlinde [7]. In this viewpoint, gravity appears as an entropic force resulted from the changes in the information associated with the positions of bodies. He derived the Newton's law of gravitation with the assumption of the entropic force together with the equipartition law of energy and the holographic principle.

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In a cosmological setup, considering the holographic principle, the energy equipartition law, the Unruh temperature with the Komar mass as the source to produce the gravitational field, one can obtain the Friedmann equations of the FRW Universe [8].

A similar approach was also implemented by Padmanabhan [9]. He obtained the Newton's law of gravitation by combining the equipartition law of energy for the horizon degrees of freedom with the thermodynamical relation $S = \frac{E}{2T}$ such that S, T and E are the entropy and the horizon temperature and the active gravitational mass, respectively [10]. He also argued that the current accelerated expansion of the Universe can be derived from the discrepancy between the surface and bulk degrees of freedom through the relation $\Delta V/\Delta t = N_{\rm sur} - N_{\rm bulk}$ such that $N_{\rm bulk}$ and $N_{\rm sur}$ are the degrees of freedom related to matter and energy content inside the bulk and surface area, respectively [11]. Note that in this way, the existence of a spacetime manifold, its metric and curvature is presumed, primarily.

These studies magnifies the importance of thermodynamics and the corresponding thermodynamical quantities even for cosmological systems. In this regard, the entropy and its bounds for thermodynamical systems are vastly investigated by various physicists. The existence of a universal bound on the entropy S of any thermodynamic system with the total energy M is proposed by Bekenstein as [12]

$$S \leq 2\pi RM, \tag{1}$$

where R is defined as the *circumferential radius* of the system as

$$R = \sqrt{\frac{A}{4\pi}}, \tag{2}$$

such that A is the area of the smallest sphere circumscribing the system. For a system contained in a spherical volume, the gravitational stability requires the condition $M \leq R/2$. Then, the equation (1) can be rewritten as

$$S \leq \frac{A}{4}. \tag{3}$$

The derivation of equation (1) involves a gedanken experiment in which a thermodynamical system is dropped into a much larger size Schwarzschild black hole. Based on the generalized second law of thermodynamics, the entropy of the system should not exceed the entropy of the radiation emitted by the black hole while relaxing to its original size [13, 14, 15, 16]. The corresponding entropy to this radiation is estimated in the works [17, 18]. This entropy bound was shown to hold in wide classes of thermodynamically equilibrium systems, independent of the fundamental derivation of the bound [19]. In order to keep the validity of these bounds, Bekenstein imposed some conditions. One can refer to these conditions as: *i*) the system must have constant and finite size, *ii*) the system must have limited self-gravity[1], *iii*) the matter components with negative energy density should not be allowed[2]. In this regard, the so-called *Bekenstein system* is a thermodynamical system which satisfies all of the mentioned conditions for the application of Bekenstein's bound. When these conditions are not satisfied, some entropic bounds can easily violate the Bekenstein's bound. The simplest example is a homogeneous spacelike hypersurface in a flat Friedmann-Robertson-Walker Universe. The entropy of a sufficiently large spherical volume will exceed the boundary area [20]. This is because space is infinite, the entropy density is constant, and volume grows faster than surface area. As another example, consider a system undergoing gravitational collapse. Before it is collapsed towards the black hole singularity, its surface area becomes arbitrarily small. Since the entropy cannot decrease, the Bekenstein's bound is violated. From the point of view of semi-classical gravity and thermodynamics, there is no reason to expect that any entropy bound applies to such systems.

Some counterexamples had been proposed in [21, 22, 23, 24]. These candidates for counterexamples to the Bekenstein's bound are clarified and refuted by Bekenstein in [25] by stressing that the energies of all essential parts of the system must be included in the energy which is imposed by the bound. Also in Ref.

[1]Here, the limited self gravity means that the gravity must not be the dominant force in the system. Consequently, one has to omit gravitationally collapsing objects and sufficiently large regions of cosmological space-times by this condition.

[2]The reason is that the bound relies on the gravitational collapse of systems with excessive entropy and is intimately connected with the idea that information requires energy. With allowing the matter with negative energy, one is able to add entropy to a system without increasing the mass, by adding enthropic matter with positive mass as well as an appropriate amount of negative mass.

[26] he refuted the two counterexamples reported in [21], while in Ref. [27] he was successful in showing that the Page's proposed bound [21] as the alternative of the bound (1) is also violated.

The holographic conjecture [28, 29], was a good starting point for Fischler and Susskind [20] to suggest some kind of entropy bounds which hold even for large regions of cosmological solutions, for which Bekenstein's conditions are not satisfied. The Fischler-Susskind bound [20], is not a general proposal. This is because, for example, it applies to the universes which are not closed or re-collapsing, while for sufficiently small surfaces in a wide class of cosmological solutions one can find other prescriptions [30, 31, 32, 33, 34].

A general proposal by Bousso [1] for entropy bound, was a successful project in imposing covariant entropy conjecture. The contribution of Bekenstein's seminal paper [12] in completing this project is undeniable. Also, the proposal of Fischler and Susskind [20], in using light-like hypersurfaces to relate entropy and area is very important. Using of light-rays, this proposal can formulate the holographic principle [28, 29, 35]. Indeed, Corley and Jacobson [35] were pioneer to take a space-time point of view in locating the entropy related to an area. The concept of "past and future screen-maps" and the suggestion of choosing only one of the two in different regions of cosmological solutions were their achievement. Moreover, they recognized the importance of caustics of the light-rays leaving a surface. The application of Bekenstein's bound to sufficiently small regions of the Universe can be found in the investigation of authors [20, 30, 31, 32, 33, 34]. They have carefully exposed the difficulties that arise when such rules are pushed beyond their range of validity [20, 30, 33]. These insights are invaluable in the search for a general prescription.

In a complement view by Bousso, in the way of proposing an entropy bound for a system, he conjectures the following entropy bound which is valid in all spacetimes admitted by Einstein's equation: Let A be the area of any two-dimensional surface. Let L be a hypersurface generated by surface-orthogonal null geodesics with non-positive expansion. Let S be the entropy on L. Then $S \leq A/4$. [1]. In this paper, we will apply the covariant entropy bound on spatial cosmological region and compare it with entropy bound coming from the Padmanabhan's Emergent Paradigm. In doing so, first of all we consider the covariant entropy conjecture in the first section. In section two, we find an entropy bound which comes by means of Padmanabhan's Emergent Paradigm. In section three, we attempt to identify the maximum entropy bound coming from covariant entropy conjecture, with the entropy of Padmanabhan's Emergent Paradigm.

2 Cosmological Entropy Bounds

2.1 Covariant Entropy Conjecture

Covariant Entropy Conjecture *Let M be a 4-dimensional space-time manifold on which Einstein's equation is satisfied subject to the dominant energy condition for the matter. Let A be the area of a connected 2-dimensional spatial surface B contained in M. Let L be a hypersurface bounded by B and generated by one of the four null congruences orthogonal to B. Let S be the total entropy contained in L. If the expansion of congruence is non-positive at every point on L (measured in the direction away from B), then $S \leq A/4$* [1].

The conjecture has been formulated so as to make the formal accuracy and generality for the entropy bound to be valid even when the Bekenstein's conditions are not satisfied. For the practical purposes, it is better to translate this entropy conjecture into a set of rules like the followings,

1. In the space-time M one can choose any two-dimensional surface B.

2. One can find four families of light-like rays $(F_1 \dots F_4)$ which have been projected orthogonally away from B, unless B is on a boundary of M .

3. It can be assumed that, without loss of generality, the expansion of F_1 has the same sign everywhere on B. If the cross-sectional area is increasing in the direction away from B (positive expansion), F_1 must not be used for an entropy/area comparison. F_1 will be allowed only for the zero or negative expansion. Repeating this test for each family, there will be at least two allowed families. In the case that the expansion vanishes in some directions, there may be as many as three or four allowed families.

4. Choose one of the allowed families, F_i. Then, construct a null hypersurface L_i, by following each light-ray until one of the following cases happens: i)The cross-sectional area spanned by the family

begins to increase in a neighborhood of the light-ray or briefly the expansion becomes positive. *ii*)The light-ray reaches a singularity or a boundary of the space-time.

The hypersurface L_i which comes by following this procedure will be called a *light-sheet* of the surface B. Different light-sheets are obtained for every allowed family.

5. The entropy conjecture represents that the entropy S_i on the light-sheet L_i will not exceed a quarter of the area of B:

$$S_i \leq \frac{A}{4}. \tag{4}$$

Note that one can apply the bound to each light-sheet individually. Because B can have up to four light-sheets then, the total entropy on all light-sheets could add up to as much as A.

Adding some remarks on the points 2 and 3 are useful. In many situations it is suitable to name $F_1 \ldots F_4$ the future-directed ingoing, future-directed outgoing, past-directed ingoing and past-directed outgoing family of surface-orthogonal geodesics. "ingoing" and "outgoing" are just arbitrary labels. Distinguishing the "ingoing" and "outgoing' rays is meaningful if B is closed.

Since the conjecture is manifestly T- invariant (time reversal invariant), the covariant entropy bound does not even refer to "future" and "past.' This is the most significant property of covariant entropy conjecture. The other point is that, the thermodynamic entropy and the generalized second law of thermodynamics, which underlies Bekenstein's bound, are not T- invariant. So one can say that the spacelike projection theorem is not T-invariant. This means that it refers to past and future explicitly. This property persuaded some people to conclude that the origin of the covariant bound is statistical rather than thermodynamics[1].

2.2 Cosmological Entropy Bound via Covariant Entropy Conjecture

The covariant entropy conjecture is a correct law, which results in an entropy bound for spatial regions in cosmology.

An interesting application of the spacelike projection theorem is for the normal regions [1]. It shows that in which situations we can treat the interior of the apparent horizon as a Bekenstein system. Let A be the area of a sphere B. It can be on or inside of the apparent horizon (the word "inside" has a natural meaning in normal regions). In this condition the future-directed ingoing light-sheet L exists, so we can assume that B is complete (B is the only boundary of apparent horizon)[3]. Let V be a region inside of and bounded by B, on any spacelike hypersurface containing B. If no black holes are produced, V will be in the causal past of L, and the conditions for the spacelike projection theorem are satisfied. Therefore, the entropy on V will not exceed $A/4$. In particular, we may choose B to be on the apparent horizon, and V to be on the spacelike slice preferred by the homogeneity of the FRW cosmologies. We summarize these considerations in the following corollary:

Cosmological Corollary *Let V be a spatial region inside the apparent horizon of an observer. If the future-directed ingoing light-sheet L of the apparent horizon has no other boundaries and if V is entirely contained in the causal past of the light-sheet L, the entropy on the spatial region V cannot exceed a quarter of the area of apparent horizon* [1].

According to above explanations, the spheres beyond the apparent horizon are anti-trapped and do not possess future-directed light-sheets. So the spacelike projection theorem cannot be applied, and the statement about the entropy enclosed in the spatial volumes cannot be made. Thus, the covariant entropy conjecture considers the apparent horizon as a particular surface. It indicates the largest surface to which the spacelike projection theorem can possibly apply, and hence the largest region one can apply as the Bekenstein's system is the inside of it.

The de Sitter space is an example in which the conditions of above corollary are satisfied by its apparent horizon. Here, the cosmological horizon is the same as apparent one at $r = (3/\Lambda)^{1/2}$. So, with the above explanations the entropy within the cosmological horizon cannot exceed $3\pi/\Lambda$.

[3]This condition is fulfilled, e.g., for any radiation or dust dominated FRW Universe with no cosmological constant.

Bekenstein's bound can be applied to spatial regions in cosmological solutions with the use of this corollary. It can be derived by the covariant entropy bound but it is not equivalent to it. This corollary is a statement of limited scope like as the spacelike projection theorem [1]. It does not give any information about relation of entropy with the area of trapped or anti-trapped surfaces in the Universe. Even for the surfaces contained in the apparent horizon, a "spacelike" bound applies only under some certain conditions. Thus, this corollary is used to define the range of validity of Bekenstein's entropy/area bound in cosmological solutions. Moreover, the covariant entropy conjecture associates at least two hypersurfaces with any surface in any space-time, and bounds the entropy on those hypersurfaces. For this reason, the corollary is far less general than the covariant entropy conjecture.

3 The Entropy Bound of Emergent Paradigm

According to Padmanabhan's proposal, the difference between the surface degrees of freedom and the bulk degrees of freedom in a region of space may result in the accelerated expansion of the Universe through the relation $\Delta V/\Delta t = N_{\rm sur} - N_{\rm bulk}$ where $N_{\rm bulk}$ and $N_{\rm sur}$ are referred to the degrees of freedom related to matter and energy content inside the bulk and surface area, respectively [11].

For an expanding Universe, we have the following condition for the Padmanabhan's formula

$$\frac{\Delta V}{\Delta t} \geqslant 0, \tag{5}$$

which demands

$$N_{\rm sur} - N_{\rm bulk} \geqslant 0. \tag{6}$$

On the other hand, we know that the relation between surface entropy and surface degrees of the freedom is as follows

$$4S = N_{sur}, \tag{7}$$

where the entropy of the surface is $\frac{A}{4}$, A being the area of the surface enclosed by the Hubble horizon.

One can also write the bulk degrees of freedom in terms of its energy and temperature as

$$N_{bulk} = \frac{2E}{T}, \tag{8}$$

where the thermodynamic temperature of our cosmological system is $\frac{H}{2\pi}$. So, one can rewrite the equations (6), (7) and (8) as follows

$$\pi r_H E \leqslant S. \tag{9}$$

This relation gives a lower bound for the entropy of a cosmological system in the framework of emergent Universe scenario.

4 Covariant Entropy Bound in Emergent Model

4.1 Misner-Sharp Energy

In this section, we consider the covariant entropy bound and the entropy bound which comes by means of Padmanabhan's formula in the emergent model, leading to the equation (9), and compare them with each other. Let us start with the Misner- Sharp energy. One can calculate the total Misner-Sharp energy inside the Hubble horizon as

$$M(r_H) = \int_0^{r_H} 4\pi r^2 \rho dr = \frac{4\pi}{3} r_H^3 \rho, \tag{10}$$

where r_H is the Hubble horizon radius and $M = E$. Moreover, for the apparent horizon we have $r = 2M(r)$ in which for our cosmological case with a flat spatial geometry the apparent and Hubble horizons coincides and consequently this formula takes the form of $r_H = 2M(r_H)$. Also, using the Friedmann equations for $k = 0$, we have $r_H = \sqrt{\frac{3}{8\pi\rho}}$. Then, using the equations (9) and (10), we obtain

$$\frac{\pi r_H^2}{2} \leqslant S, \tag{11}$$

where the equality holds for the horizon r_H. So, the mass-saturated Bekenstein bound, i.e for $r_H = 2M(r_H)$, will be

$$S_{max} = \pi r_H^2, \tag{12}$$

which is exactly the maxima of the covariant entropy bound $S \leq \frac{A}{4}$, i.e $S = \frac{A}{4} = \pi r_H^2$.

On the other hand, one can treat the inside region of the Hubble horizon as a Bekenstein system which surrounded by Hubble horizon surface possessing the entropy/mass bound $S \leq 2\pi M r_H$, where S denotes its internal entropy. Then, by using the equations (9) and (12), we obtain the following total bound on the entropy

$$\pi r_H E \leqslant S \leqslant \pi r_H^2. \tag{13}$$

Recalling that both equality cases happen for the Hubble horizon, we arrive at an inconsistency between covariant entropy bound and the bound coming from Padmanabhan's formula, because of the Misner-Sharp energy $E = \frac{r_H}{2}$ on r_H.

4.2 Komar Energy

In this subsection, we repeat the calculation of the pervious subsection for the Komar energy and try to remove the above inconsistency. To begin with, we consider the Komar energy as the total energy in the bulk enclosed by the surface of the Hubble horizon, as

$$E(r_H) = \int_0^{r_H} 4\pi r^2 |\rho + 3p| dr = \frac{4\pi}{3} r_H^3 \rho |1 + 3\omega|, \tag{14}$$

where we have considered the barotropic equation $p = \omega\rho$. Then, from the inequality (9), we obtain

$$4\pi r_H (\frac{4\pi}{3} r_H^3 \rho)|1 + 3\omega| \leqslant A = 4S, \tag{15}$$

where the L.H.S becomes maximum (equality case) at r_H as

$$A = 4S_{max} = 2\pi r_H (2M(r_H))|1 + 3\omega|. \tag{16}$$

Then, using the equation (9) and covariant entropy bound, we obtain

$$\frac{\pi r_H^2 |1 + 3\omega|}{2} \leqslant S \leqslant \frac{A}{4} = \pi r_H^2. \tag{17}$$

5 Conclusion

By applying Bousso's covariant entropy conjecture for the cosmological spatial region in one hand, and the entropy bound which comes from the Padmanabhan's Emergent Paradigm, on the other hand, we have obtained the relation (17) with interesting results. The first interesting result is that other than the upper entropy bound imposed by Bousso's covariant entropy conjecture for the cosmological spatial region, there is a lower entropy bound obtained through the Padmanabhan's Emergent Paradigm. The second interesting result is that the lower entropy bound coincides with the upper entropy bound, as the maximal entropy, provided that: *i*) inside of the apparent horizon be filled by the radiation, namely $\omega = \frac{1}{3}$ or *ii*) inside of the apparent horizon be pure de Sitter space subject to the cosmological constant, namely $\omega = -1$. In other words, the maximal entropy inside the apparent horizon occurs when it is filled completely by the radiation field or cosmological constant. In both cases the holographic principle is satisfied in the sense that the number of degrees of freedom in the bulk becomes equal to the number of degrees of freedom on the surface. The fact that both radiation and cosmological constant correspond to the maximal entropy inside the apparent horizon, may represent a duality symmetry between the radiation and cosmological constant from the holographic principle point of view. The origin of this duality symmetry may be a "one to one" correspondence between the number of degrees of freedom in the bulk and on the surface. At early Universe, dominated by the cosmological constant, the number of degrees of freedom in the bulk and on the surface are equal. At the subsequent radiation dominant era, the correspondence between the number of degrees of

freedom in the bulk and on the surface still holds. In other words, the transition from cosmological constant to radiation dominant eras does not alter the maximal entropy property of the universe. However, at matter dominant era $\omega = 0$ the maximal entropy property is broken in the sense that the number of degrees of freedom in the bulk becomes smaller than the number of degrees of freedom on the surface. The decrease in the number of degrees of freedom in the bulk, in comparison to the enclosed surface, which accounts for the breakdown of "one to one" correspondence may be justified by the structure formation in the bulk at matter dominant era. In other words, the structure formation in the bulk can reasonably decrease the number of degrees of freedom (and the entropy) in the bulk in comparison to the surface. Therefore, one may interpret the current acceleration of the Universe as a tendency of the system of Universe to recover the maximal entropy property at late time (dominated by cosmological constant with $\omega = -1$) which was already lost at matter dominant era ($\omega = 0$).

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